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Developing a Hierarchical Fuzzy Rule-based Model with Weighted Linguistic Rules: A Case Study of Water Pipes Condition Prediction

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***Abstract*—Assessing the condition of water pipes is a complex task, partly due to scarcity of complete maintenance records and field observations. This makes it harder to identify the factors determining pipe condition and their probabilistic relationships with the deterioration process. A challenge facing water utilities is to find an effective and reliable tool for assessing their pipelines and taking prompt decisions regarding repair and maintenance to extend the service life and keep them safe from sudden failures.**

This paper presents research on a new fuzzy-based methodology for modelling water pipe condition prediction. It proposes a hierarchical fuzzy rule-based model that uses a simplified and effective method for supporting the elicitation of the fuzzy rules and adapting uncertainty propagation that can be intuitively understood by human experts. The results of applying the model to the water pipes domain shows the plausibility of extending the approach to other knowledge domains based on human expertise.

Keywords*—Fuzzy Inference; Fuzzy Modelling; Fuzzy Numbers; Hierarchical Fuzzy Rule-based Model; Water Pipe Condition Prediction

I. Introduction

Buried water pipes can be made of many materials, such as cast iron, ductile iron, asbestos cement, polyvinyl chloride, and pre-stressed concrete cylinder. It is vital to keep evaluating their structural integrity and performance because they inevitably degrade. For example, in a recent assessment study of America's drinking water infrastructure, the American Society of Civil Engineers (ASCE) assigned it a grade of poor, "D". It indicated that much of drinking water infrastructure is nearing the end of its service life and facing risk of failure. The cost of replacing all the water pipes was estimated to be more than 1 trillion US-dollars [1].

All buried water pipes are prone to failure under normal service conditions. The deterioration and failure process is complex and depends heavily on pipe material, environmental surroundings, and operational conditions [2]. Kleiner and Rajani [3] described the buried pipes lifecycle in three phases. The first phase, known as "burn-in", describes the period immediately after the installation where failure can occur due to manufacturing defects and storage, and improper construction processes. The second "in-usage" phase is when the pipe begins service. Failure in this phase may occur due to inappropriate maintenance, natural hazards, and external interference. The third phase is "wear-out", in which the probability of failure is increased due to deterioration and ageing.

To date, there is no prescribed method for water utilities to assess their buried pipes. Alternative approaches include destructive testing and nondestructive testing [4], [5], [6]. The field observations obtained from inspections are related to the condition of pipes and converted into an overall condition rating so that an appropriate course of action regarding repair and maintenance is undertaken. However, obtaining such observed data is not always feasible due to the prohibitive costs of applying direct inspections and/or inability to apply them while the pipeline is operating. In these situations, knowledge from experienced experts can be exploited to model the relationships between factors (inputs) and condition of pipe (output).

The aim of the research presented in this paper is to explore how different types of data and variations in the degree of uncertainty within a system can be modelled using fuzzy methods and how these methods can both complement and enhance human expertise. The research will primarily focus on studying and analyzing the large diameter Pre-stressed Concrete Cylinder Pipe (PCCP) type. It is widely used as the transmission part (backbone) of water supply systems around the world due to its high capability for resisting large internal pressure and external forces [7]. The problems of analyzing the pipe domain are not unique and the research is motivated to produce a generic methodology that can be applied in alternative domains.

The paper will firstly provide a review of general approaches used in modelling the failure and condition rates of water pipes. This will help identify the performance and capabilities of the developed models and hence the barriers facing the development of an effective and reliable model. An overview of using the fuzzy approach in water pipe condition modeling is then provided, with a focus on hierarchical models. The rationale for the proposed fuzzy model is discussed, explaining how it tackles the limitations of previous models. The detailed steps of model construction and information processing are then described with given examples. The results of applying the model to real-world PCCP data are provided and the model performance is discussed. The paper ends with

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a discussion of the advantages of the proposed model and the next steps to be taken.

II. Water Pipe Condition and Failure Prediction Approaches

In general, there are different types of modelling approaches in the literature used for predicting the condition of water mains. These approaches can generally be classified into three categories: (1) statistical, (2) probabilistic and (3) artificial intelligence (AI). The condition and/or performance in small water distribution mains is assessed through the observed failure rate (frequency), in large water transmission mains it is usually through a condition rating scale due to the scarcity of the occurrence of failure events [3], [5].

The statistical modelling approach is widely used in solving engineering problems. For water mains, historical data on past inspections and failures have to be available in order to determine the mechanism and rates that can be assumed to continue in future. It is usually applied to small and medium distribution water mains where recorded historical failures and/or condition data are much easier to obtain [3]. These methods can be classified into survival analysis models that use Weibull/Exponential equations [8], [9], regression models [10], [11], and simple time-linear and time-exponential models [12], [13]. Generally, they are simple mathematical models and can be used for any type of pipe materials.

A limitation of statistical modelling is the requirement for large amounts of historical pipe condition/failure data recorded over a long period of time, which limits the variety of variables available for analysis. Kleiner et al. [14] argues that models should include as many independent variables as possible. Using few measurable (objective) pipe factors (e.g. age, length, diameter, breakage history) without including other important operational and environmental time-dependent and qualitative factors will not properly account for the subjective and probabilistic nature of pipe deterioration and condition. Another limitation of such models is that they do not provide information about pipes that have not yet failed; one of the motivations for the present work is to produce a model capable of inferring the condition of pipes that do not indicate any signs of distress.

The probabilistic modeling approach utilizes the probability or relative frequency distributions of certain input values to predict distributions or ranges of output values. It has been used to predict the failure rates of pipelines in general [15], [16] and the failure risk of water pipelines in particular [17], [18]. However, they are usually applied where the interrelationships between the factors and process of deterioration and failure are well understood; otherwise, large data sets are needed to train and understand relationships between factors.

Applying artificial intelligence (AI) techniques also depends on the type and availability of data. When large amounts of historical data exist, a data-driven approach, such as artificial neural networks (ANN), can be used to determine model structure and learn cause-effect relationships and uncertainties. Such models have the ability to learn and generalise results over time [18], [19], [20] and have recently been used for modelling the condition and deterioration of water pipe infrastructures [21], [22], [23], [24].

Case-based reasoning (CBR) techniques can be applied when a large and variant database of experienced cases (case library) about the problem is available [25], [26]. In CBR, a specific output (e.g. pipe condition) given a set of input data can be predicted by retrieving and adjusting relevant information obtained from the case library. In this approach, the case library has to be updated in order to obtain accurate results [27]. Fuzzy-based approaches come into their own when historical data are not available or, if available, they are ambiguous or imprecise. The missing knowledge comes from human expertse in the specific field. Fuzzy systems usually provide a tool, such as rules, for representing the knowledge base of the system that allows fusion and aggregation of imprecise (vague) information/data throughout the components of the system [28].

The theory of fuzzy sets was developed in 1965 by Zadeh [29] to provide a tool for dealing with the imprecision inherent in many problems, and for representing uncertainty and vagueness that cannot be represented by conventional crisp sets methods. Fuzzy sets are based on a membership function that has a continuous grade varying between 0 and 1. Each value within the range of the fuzzy set has a membership grade determining how much it belongs to the set. Fuzzy membership functions can be efficiently used to deal with the imprecise input data and their ranges (categories).

Fuzzy sets are suited to knowledge systems represented as if-then rule statements, which have the advantage of describing the system linguistically [28], [30], [31]. The fuzzy-based approach has been widely applied to infrastructure management including modelling water pipe condition assessment [32], [33], [34], [35].

III. Fuzzy Modelling Approach

A. Hierarchical Fuzzy Rule-based Approach

One of the main issues in fuzzy modelling is the structuring of data, which can have a significant effect on the performance of the model. A good understanding of the pipe lifecycle, process and causes of failure together with the nature and characteristics of data will help arrive at a meaningful and reasonable structure. In this research, a hierarchical fuzzy rule-based approach is adopted to model the condition prediction of buried PCCP water pipes. This approach is highly suited to accommodating and integrating uncertainties inherent in the problem due to the imprecision of measurements of objective data (e.g. relative reliability of inspection/testing methods and human judgement of results). The existence of qualitative data that are linguistically described and the vagueness of the relationships between the data also lend themselves to a fuzzy approach. Furthermore, they support representations that resonate with how experts conceptualise the problem. This facilitates knowledge elicitation, which can be done explicitly using rules consisting of linguistic terms that the experts understand; it is an advantage of fuzzy rule-based approaches over other methods used in decision support systems [28], [36].

One reason for a hierarchical fuzzy approach is to overcome the curse of dimensionality [37], [38]. In standard fuzzy systems, there is an exponential growth in the number of rules as the number of input variables increases. Suppose n is the number of input variables and m is the number of linguistic quantifiers for each variable, then the total number of fuzzy rules is m^n for every combination of quantifiers. For an example of n=6 and m=2 the number of rules required is 2^6=64. If the system is decomposed into 2 fuzzy sub-units, each with 3 variables, and a sub-unit of output aggregation, then the total number of rules is the sum of all fuzzy sub-unit rules ($2^3+2^3+2^2$=20). In this way, the total number of fuzzy rules is reduced significantly to a reasonable and manageable level, which increases system transparency and interpretation.

B. Previous Hierarchical Fuzzy Models

The hierarchical fuzzy approach has been applied to several domains for modelling risk prediction and condition assessment. Among relevant and interesting models are translating inspection results into condition ratings for water pipes [34], evaluating the risk of water main failure [35], assessing risk in the mining industry [39], and railway risk analysis [40].

In Rajani et al. [34], observations from visual inspection and non-destructive tests are converted into water main condition ratings using a fuzzy approach, called "*fuzzy synthetic evaluation technique*". Data were organised into a two-level structure (i.e. indicators and categories), and then evaluated based on three-steps: *fuzzification*, *aggregation* and *defuzzification*. The aggregate effect of inputs at each level is calculated by synthesising their weights and fuzzy membership values using matrix multiplication.

Fares and Zayed [35] incorporated 16 risk of failure factors grouped into four sub-models (environmental, physical, operational and post failure) and another sub-model that combines the results of the previous ones to produce a risk of failure. The model uses a fuzzy rule-based inference method to calculate the risk of failure in a crisp format.

Shikha and Sharad [39] and An et al. [40] used a fuzzy reasoning approach to calculate the risk level of each hazardous event with respect to three factors. They employed a fuzzy analytical hierarchy process technique to determine the relative weights of the risk factors so that the risk assessment is progressing from the base level to the final system level by synthesising factors and their weights at each level.

Based on this review, certain issues emerged with respect to data structuring and knowledge processing using a hierarchical fuzzy approach. The way the model is constructed should appropriately account for modelling vague relationships between variables and how they interact with each other so that dissonant data are not in the same category. This may affect knowledge processing leading to discrepant or contradictory results. It may explain why Fares and Zayed's model [35] found that pipe age has the highest impact on water main risk of failure followed by pipe material and breakage rate. These results contradict received wisdom in the water pipe domain that the direct distress factors (e.g. breakage, cracks, joint leaks) have more effect on pipe failure process than indirect distress factors (e.g. age, pressure rate, soil conditions).

Pipes can continue service for even longer than their design life if operational and environmental conditions are stabilized but a breakage means the presence of a leak, which indicates that the failure has already started. It is preferable to include as many varied and independent variables as possible in modelling so that the model can be generalised for evaluating the entire set of pipe situations. Furthermore, the models reviewed used different types of defuzzification methods to transform fuzzy values into crisp values at different stages of knowledge processing, which causes some information loss. For example, the information lost in defuzzifying the lower level sub-model will be propagated to the upper level sub-model in the system, which affects the accuracy of the outcomes [38]. This is compounded with an increasing number of hierarchical levels in the system.

This paper describes a new fuzzy-based methodology to tackle these issues. The suggested improvements are as follows.

- A tractable method for generating the set of weighted fuzzy rules by using fuzzy processing of the variable relative influence and the impact of each variable value on the associated output. It utilises fuzzy numbers in the process of transforming and combining fuzzy values (regarding variable weights and their impact on the output) [41] such that full information about uncertainty is maintained throughout the process.
- Incorporating the fuzzy weights of input variables into the rule so that they carry through to the inference network and allow a better way for adapting uncertainty propagation throughout the system components.
- Delaying defuzzification of fuzzy values in the inference process until the very end of the process so that uncertainty information is maintained during knowledge propagation (i.e. once input variables are fuzzified at the first-level, information is propagated in its fuzzy format throughout the hierarchy to preserve the entire fuzzy data).
- A new structuring of condition data that utilizes both direct and indirect distress data organized into concepts: each variable in the first level contributes towards the condition within its concept (group) and is the same for the concepts in subsequent levels. Therefore, the overall condition of the pipe is calculated by aggregating the interaction of these variables and concepts throughout the hierarchy structure.

C. Fuzzy Numbers

Fuzzy numbers are defined as a fuzzy subset of a universe of discourse on real numbers R [42], [43]. They allow better representation of linguistic/inexact variables than ordinary numbers and provide a suitable means of transforming a fuzzy (vague) environment into a mathematical model. In the proposed fuzzy methodology, experts state their opinion linguistically regarding the comparisons of variables' importance. In addition, values associated with inputs and output are also linguistically represented. A triangular fuzzy number is used to express these vague values. It is fast in computation and gives more intuitive and natural interpretation due to the simple shape of its membership function [44]. Generally, a triangular fuzzy number, a, is represented by three points: $a = (l, m, u)$, where l and u are the lower and upper bound of the fuzzy number, respectively, and m is the median value. The membership function of the triangular fuzzy number, $\mu_a(x)$, is shown in Figure 1.

When used in judgment, the interval of a triangular fuzzy number, (l, m, u), could be interpreted as the minimum possible value, the most possible value and the maximum possible value, respectively.

$$\mu_a(x) = \begin{cases} \frac{x-l}{m-l}, & for\ l \le x \le m \\ \frac{u-x}{u-m}, & for\ m \le x \le u \\ 0, & for\ otherwise \end{cases}$$

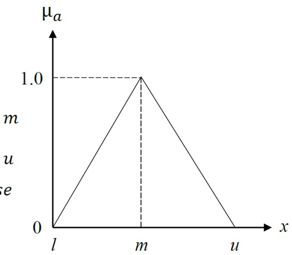

Fig. 1. Triangular fuzzy number, $a = (l, m, u)$.

D. *Fuzzy Analytic Hierarchy Process*

The Analytic Hierarchy Process (AHP) was introduced by Saaty [45] and has been widely used as an analysis and evaluation tool for solving many practical multi-criteria decision making problems. In its traditional formulation, experts' judgments are represented as exact numbers (ratios) to form the comparison matrix for criteria and alternatives. Buckley [46] has fuzzified Saaty's AHP by employing fuzzy ratios instead of exact ratios in representing preference and personal judgment in order to handle the inherent uncertainty and imprecision associated with many problem domains in reality. The method has become known as the Fuzzy Analytic Hierarchy Process (FAHP) and many researchers have used it in various domains, including service evaluation [47], vulnerability of cities with respect to earthquakes [48], hospital organisation [49], locating logistics centers for disasters [50], risk of hazardous materials transportation [51], power-plan risks [52], and risks for assembling satellites [53].

Saaty and Tran [54] explicitly criticised the fuzzification of the AHP method and stated that there is already uncertainty inherent in the nature of the method: i.e. the comparison judgements are already fuzzy because they are allowed to vary over the values of a scale. However, comparisons have been done for AHP and FAHP by comparing their results with those of experts and, in general, using FAHP leads to better model outcomes than the AHP. More discussion on this issue is provided later in the paper. For now, our research has employed the FAHP within a new framework to build the set of fuzzy rules.

IV. Proposed Fuzzy Model

As systems modelling is an abstraction (or simplification) of reality, it is important to consider the different types of uncertainty that may be involved in the modelling process and how they can be represented. Improper identification and/or estimation of uncertainty will reduce the usefulness of model outcomes as well as the power of the knowledge embedded in the model [55]. According to Walker et al. [56], uncertainty can be due to lack of knowledge (epistemic uncertainty) or due to natural variability (variability uncertainty) inherent in the system under study. In our problem, condition prediction of PCCP water pipes, uncertainty may come from the inherent resolution of the testing technology, the natural variability of input data (time-dependent), the imprecision of natural language that measures qualitative data, the interpretation of test signals converted into quantitative numbers and experts intuitive knowledge of formulating the relationships between variables. Fuzzy sets and fuzzy numbers based on linguistic variables are therefore an appropriate approach.

A. *Model Construction*

PCCP condition assessment data are organized in a three-level structure, as shown in Figure 2. It is designed to consider the type and possible effect of the condition data so that it better reflects the variable relationships and their interaction.

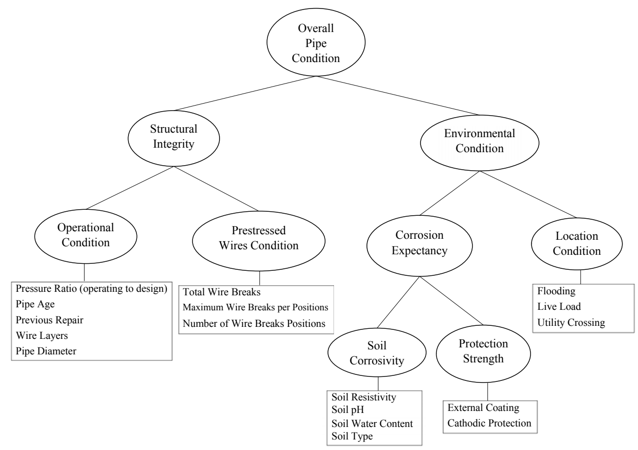

Fig. 2. PCCP condition prediction model structure where data are organized into concepts interacting together towards the overall pipe condition.

We select the concept *Prestressed Wires Condition (PWC)* with the variables *Total Wire Breaks (TWB)*, *Maximum Wire Breaks per Positions (MWBpP)* and *Number of Wire Breaks Positions (WBP)* to explain the proposed fuzzy methodology (i.e. fuzzy rules construction and fuzzy inference process). Both the concept and variables are assigned linguistic values/labels represented by triangular membership functions. The membership functions for the concept and variables are derived from experience and empirical studies. Figure 3 shows the membership function for *PWC* and *TWB*.

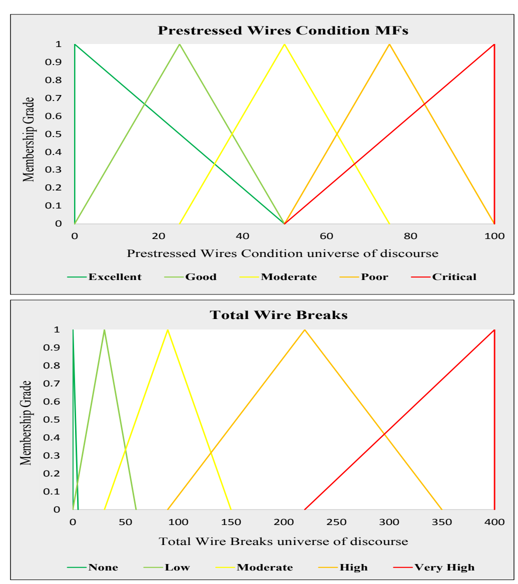

Fig. 3. Membership functions for the concept *PWC* and the variable *TWB*.

The knowledge base of the proposed model is represented by a set of If-Then fuzzy rules illustrating the relationship between inputs and output. An example of the fuzzy rules structure in a symbolic form is given next, where x_i is the i^{th} input variable (i=1, 2, ..., n), y is the output variable, A_{ik} is the linguistic value (fuzzy set) of the i^{th} input variable in k^{th} rule (k=1, 2, ..., r), B_k is the linguistic value (consequent fuzzy set) of the output variable in k^{th} rule, n is the number of input variables, and r is the number of rules.

IF x_1 is A_{1k} AND x_2 is A_{2k} ..... AND x_n is A_{nk} THEN y is B_k

1) Fuzzy Rules Construction Algorithm:

Step 1: Determine all possible combinations of variables with their different values. In our example, we have three variables that contribute to a concept (two variables with five linguistic values and the other with four linguistic values). Then there will be 100 combinations of variable values (5×5×4) that represents the entire relationship between variables, as shown in Figure 4. Each combination of variables's values represents the antecedent of the rule and their collective impact is computed and then fuzzified to determine the consequent (output linguistic value) of the rule. In this way, 100 rules will be generated to calculate this concept.

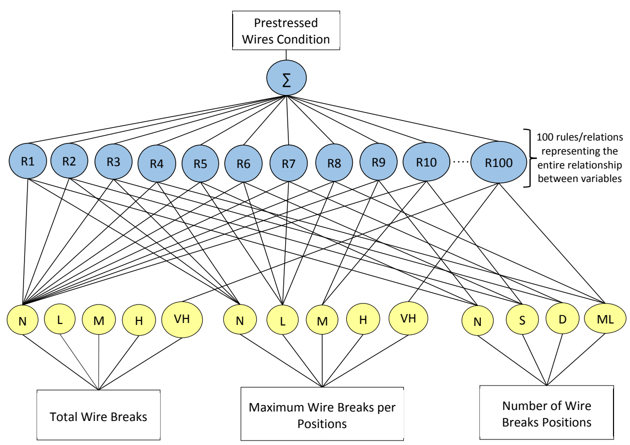

Fig. 4. Variable combinations for concept *PWC*. The yellow oval shapes are the linguistic values associated with each variable, and their acronyms shown are: N=None, L=Low, M=Moderate, H=High, VH=Very High, S=Single, D=Double and ML=Multiple. The blue oval shapes are the aggregate effect of variable combination on the concept *PWC* that needs to be calculated.

Step 2: Compute variables' fuzzy weights:

a) Obtaining pairwise comparison matrix: First, we construct a matrix that the rows and columns contain the same variables in the same order. Then, for each row, we ask the experts to compare the variable in the row with respect to each variable in the columns and express their opinion linguistically according to the fuzzy comparison scale shown in Figure 5, which is an extension to the crisp comparison scale originally developed by Saaty [45]. Only the right upper half of the matrix is filled directly by experts' judgement, while the other half is filled by taking the reciprocal values. For example, if comparing TWB to MWBpP was given a relative importance of (4,5,6), then comparing MWBpP to TWB has to be given a relative importance of (1/6,1/5,1/4). Because we have three variables in our example, we need to ask the expert to answer three pairs of comparisons. The number of pairwise comparisons needed is calculated based on this equation: *n(n-1)/2*, where n is the number of variables. An example of the questions asked is "*Which of the variables, Total Wire Breaks and Maximum Wire Breaks per Positions, is the most important for assessing Prestressed Wires Condition and how does its importance compare to the other?*". In our example, the pairwise comparison matrix of the variables is shown in Table I.

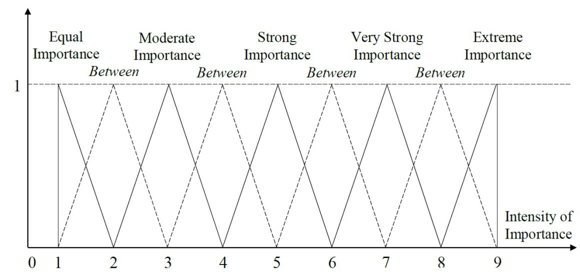

Fig. 5. Fuzzy comparison scale between variables.

TABLE I. Pairwise Comparison Matrix of the Variables

Variable	TWB	MWBpP	WBP
TWB	(1,1,1)	(1,1,2)	(4,5,6)
MWBpP	(1/2,1,1)	(1,1,1)	(4,5,6)
WBP	(1/6,1/5,1/4)	(1/6,1/5,1/4)	(1,1,1)

Csutora and Buckley [57] proved that the fuzzy pairwise comparison matrix is considered consistent if the corresponding crisp pairwise comparison matrix is consistent. The crisp format of the fuzzy pairwise comparison matrix shown in Table I is constructed, by considering the median value of the triangular fuzzy number, and the consistency ratio is calculated based on λ_{max} procedure (for more details the reader is referred to [45]). It has been found that the crisp format of the pairwise comparison matrix is consistent which means that the fuzzy pairwise comparison matrix is also consistent.

b) Compute normalized fuzzy weights of variables: There are several methods for estimating the normalized fuzzy weights. Three of them have been applied to the fuzzy pairwise comparison data: row means with geometric normalization; row means with columns sum normalization; and geometric means with fuzzy division normalization. In order to select the most appropriate method for representing data, the *mean relative fuzziness* for pairwise comparison data and resulting weights are calculated and compared [58]. Equations (1) and (2) show how the *mean relative fuzziness* is calculated.

$$FuD = \frac{1}{n^2} \sum_{i,j=1}^{n} \frac{u_{ij} - l_{ij}}{m_{ij}} \tag{1}$$

$$FuW = \frac{1}{n} \sum_{i=1}^{n} \frac{u_i^w - l_i^w}{m_i^w} \tag{2}$$

Where FuD is the mean relative fuzziness of data, l_{ij}, m_{ij} and u_{ij} are the lower-bound, the median and the upper-bound of the triangular fuzzy ratio of variables i and j, respectively, FuW is the mean relative fuzziness of resulting weights, l_i^w, m_i^w and u_i^w are the lower-bound, the median and the upper-bound of the triangular fuzzy weight of variable i, respectively.

$$G = (l_{ij}^G, m_{ij}^G, u_{ij}^G) = (\frac{l_{ij}^G}{(\sum_{i=1}^n l_{ij}^G \sum_{i=1}^n u_{ij}^G)^{1/2}}, \frac{m_{ij}^G}{\sum_{i=1}^n m_{ij}^G}, \frac{u_{ij}^G}{(\sum_{i=1}^n l_{ij}^G \sum_{i=1}^n u_{ij}^G)^{1/2}}) \quad (3)$$

$$FW_i = (l_i^{FW}, m_i^{FW}, u_i^{FW}) = (\frac{\sum_{j=1}^n l_{ij}^G}{n}, \frac{\sum_{j=1}^n m_{ij}^G}{n}, \frac{\sum_{j=1}^n u_{ij}^G}{n}) \quad (4)$$

$$CI_k = (l_k^{CI}, m_k^{CI}, u_k^{CI}) = (\sum_{i=1}^n l_i^{FW} \times l_i^{VI}, \sum_{i=1}^n m_i^{FW} \times m_i^{VI}, \sum_{i=1}^n u_i^{FW} \times u_i^{VI}) \quad (5)$$

Applying equations (1) and (2), the FuD is 0.348 and FuW is 0.337, 0.707, and 0.708 for the methods *row means with geometric normalization*, *row means with columns sum normalization* and *geometric means with fuzzy division normalization* respectively. It is noted that the comparison data and the row means with geometric normalization method resulted in almost the same relative fuzziness which indicating that this method is the most appropriate for representing the fuzzy pairwise comparison ratios, and accordingly it has been selected for application in rule construction. A graphical comparison of FuD and FuW for the entire model structure is presented in Section V.

Equations (3) and (4) show how the normalized fuzzy weights are calculated using the selected method, and Table II shows the computed normalized fuzzy weights for our example, where G is a column-normalized matrix using geometric fuzzy normalization and FW is the normalized triangular fuzzy weights.

TABLE II. Normalized Fuzzy Weights of the Variables on the Concept *PWC*.

Variable	Normalized Fuzzy Weight
TWB	(0.421,0.455,0.608)
MWBpP	(0.335,0.445,0.483)
MWB	(0.080,0.091,0.105)

The resulting fuzzy weights have also been checked for normalization and they satisfy the conditions of fuzzy numbers normalization. For more details about normalization of fuzzy numbers, see [58], [59], [60].

Step 3: Experts are asked to express their opinion regarding the effect of each value of an individual variable on the output concept as a linguistic value according to a fuzzy condition assessment scale. In our example, it is equivalent to the fuzzy membership function scale of the concept *PWC* shown in Figure 3. A sample of the questions asked is "*What would be the potential impact of Low number of Wire Breaks on Prestressed Wires Condition?*".The number of questions to be asked to the experts are equal to the total number of variables values. In our example, we need to ask 14 questions. The linguistic values obtained from experts are then converted into triangular fuzzy numbers, as shown in Table III.

Step 4: Obtain the collective fuzzy impact for each combination of variables values by multiplying the variables fuzzy weight (obtained in step 2) and their fuzzy impact (obtained in step 3). The aggregation of the multiplication would result in a collective fuzzy impact of the variables' combination on the concept, which is represented as a triangular fuzzy number. This is done using equation (5), where

TABLE III. Impact of Variables' Values on the Concept *PWC*.

Variable	Value	Impact on PWC	Fuzzy Number
TWB	None	Excellent	(0,0,50)
	Low	Good	(0,25,50)
	Moderate	Moderate	(25,50,75)
	High	Poor	(50,75,100)
	Very High	Critical	(50,100,100)
MWBpP	None	Excellent	(0,0,50)
	Low	Good	(0,25,50)
	Moderate	Moderate	(25,50,75)
	High	Poor	(50,75,100)
	Very High	Critical	(50,100,100)
WBP	None	Excellent	(0,0,50)
	Single	Moderate	(25,50,75)
	Double	Poor	(50,75,100)
	Multiple	Critical	(50,100,100)

CI_k is the collective fuzzy impact of the k^{th} combination, $VI_i = (l_i^{VI}, m_i^{VI}, u_i^{VI})$ is a triangular fuzzy number representing the impact of the variable i on the concept, and n is the number of variables in the combination.

Consider the variable combination of rule R_2 (see Figure 4) where *TWB=None*, *MWBpP=None* and *WBP=Single*. Applying equation 5 using arithmetic operations of fuzzy numbers, the resulting collective fuzzy impact $CI_2 = (l_2^{CI}, m_2^{CI}, u_2^{CI}) = (2.011, 4.545, 62.440)$.

The collective fuzzy impact, CI_k, is calculated in the same way for all variables combinations. However, in some cases the resulting collective fuzzy impact goes outside the range of possible values for the membership function scale of the concept (i.e. goes farther than 100). This is expected because the sum of the upper-bounds (u_i) of the normalized fuzzy weights of the variables is always greater than 1, which is one of the conditions of fuzzy weights normalization. In our example, the sum of variables' normalized fuzzy weights is (0.836,1.00,1.196), which is a fuzzy number centred around 1.

Step 5: Determine the linguistic value of the rule output. This is done by mapping the triangular fuzzy number of the collective impact (obtained in step 4) on the fuzzy condition assessment scale of the concept. This produces a fuzzy set of the intersections of the collective fuzzy impact and the linguistic values of the scale. The linguistic values shown in the fuzzy condition assessment scale represent different alternatives to the rule output and the membership grades of their intersection could be interpreted as the degree of preference of the alternatives. Accordingly, the resulting membership grades of the intersection for every combination of variables are ranked and the linguistic value with the highest membership grade of intersection is selected to be the output (consequent) linguistic value of the rule. Equation (6) illustrates how the highest membership grade, μ_k^{CI}, is calculated, where $\mu^{CI}(x)$ is the membership function of the collective impact, $\mu_i(x))$ is the membership function of the i^{th} linguistic value of the fuzzy condition assessment scale and s is the number of linguistic values (condition states).

$$\mu_k^{CI} = max(\bigcup_{i=1}^{s}(\mu^{CI}(x) \cap \mu_i(x))) \tag{6}$$

For illustration, Figure 6 shows the mapping of the collective fuzzy impact, CI_2, calculated in the previous step, on the fuzzy membership function scale of *PWC*. It provides a graphical interpretation to the extent of collective impact of this combination of variables on the concept *PWC*.

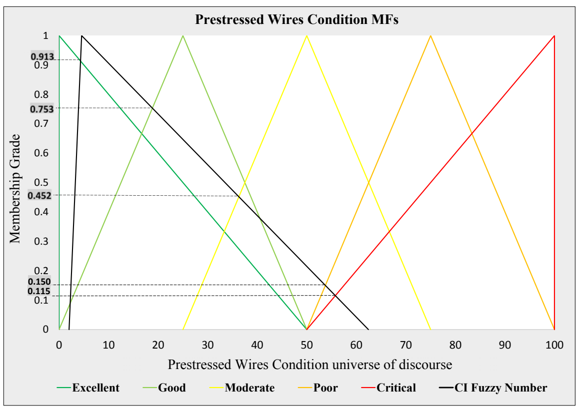

Fig. 6. Mapping of the collective fuzzy impact on the fuzzy membership function scale. The black line shows the intersection of the collective fuzzy impact and each of the condition states (linguistic values). The resulting membership grades; 0.913, 0.753, 0.452, 0.150 and 0.115 represent the degree of support of the condition states *Excellent*, *Good*, *Moderate*,*Poor*, and *Critical*, respectively, in the concept *PWC*.

Accordingly, for rule 2, the output linguistic value is selected to be *Excellent* because it holds the highest membership grade of the intersection, μ_2^{CI}=0.913. This membership grade could also be interpreted as a degree of confidence in the selected linguistic value, and thus considered as a rule weight. The mapping operation shown above is carried out for all collective fuzzy impacts of the 100 variable combinations and the fuzzy rules are then generated.

2) Fuzzy inference algorithm:

Step 1: Starting with the first level of the hierarchy structure, apply the following steps for every concept in the level.

a) Fuzzification of the input variables: The crisp (real) values of the concept's input variables are fuzzified in order to determine the membership grade associated with each linguistic value of the input variable, that is, each input is fuzzified (mapped) over all membership functions required by the rules. The membership grade, $\mu_{ij}(x_i)$, of variable i associated with the linguistic value j is determined using equation (7), where x is the crisp value that needs to be fuzzified.

$$\mu_{ij}(x) = max(min(\frac{x - a_{ij}}{b_{ij} - a_{ij}}, \frac{c_{ij} - x}{c_{ij} - b_{ij}}), 0) \tag{7}$$

b) Rule evaluation:

b.1) Antecedent evaluation: The antecedent (condition) part of the rule is evaluated to determine its firing strength (activation degree) using an appropriate fuzzy operator. The input to this operation is the membership grades for each membership function (obtained in step (a)) and the output is a single value representing firing strength of the rule. The firing strength of the k^{th} rule, α_k, is calculated using the fuzzy intersection operation shown in equation (8), where k=1, 2, 3, ...r (the number of rules), n is the number of variables.

$$\alpha_k = \bigcap_{i=1}^{n} \mu_{ik}(x_i) \tag{8}$$

b.2) Weighing the rules: All fired rules in the rule set are then weighted by multiplying their firing strength (obtained in (b.1)) and the membership grade associated with their consequent linguistic value, μ_k^{CI}, (obtained in step 5 of the fuzzy rule construction algorithm). The weighted firing strength of the k^{th} rule, α_k^{wt}, is calculated using the multiplication operation shown in equation (9).

$$\alpha_k^{wt} = \alpha_k(\cdot)\mu_k^{CI} \tag{9}$$

b.3) Consequent evaluation: The output fuzzy set of the k^{th} rule, $F_k(y)$, is obtained by reshaping (truncating) the consequent membership function of the rule using the weighted firing strength (obtained in (b.2)). This operation is illustrated in equation (10).

$$F_k(y) = \alpha_k^{wt} \bigcap \gamma_k \tag{10}$$

Where γ_k is the consequent membership function of the k^{th} rule's linguistic value.

c) Aggregation of rules outputs: All output fuzzy sets of the rules (obtained in (b.3)) are aggregated by using the fuzzy union operator in order to obtain a single fuzzy set, $F(y)$, for the output variable (concept). This operation is illustrated in equation (11).

$$F(y) = \bigcup_{k=1}^{r} F_k(y) \tag{11}$$

Step 2: Move on to the second level of the hierarchy structure and repeat step 1 by applying only sub-steps (b) and (c), i.e., the output of the first level is passed to the second level as a fuzzy value so that fuzzification of the input variables (step (a)) is no longer needed.

Step 3: If the hierarchy structure has two levels, then stop and go to step 4. Otherwise, move on to the subsequent level and repeat step 2. Continue the process until all levels of the hierarchy are evaluated. In either case, a fuzzy set of membership grades representing the pipe condition is obtained.

Step 4: Apply an appropriate defuzzification method to convert the output fuzzy set into a crisp format so that the overall condition of the pipe can either be represented in a fuzzy or crisp value.

An example of the inference network that illustrates how the knowledge is processed to estimate the condition of prestressed wires is shown in Figure 7. At first, real-world data are entered into the model and then translated into fuzzy membership grades (MG) having a value from 0 to 1. These MGs represent the variable's degree of support in its states. In the example presented (Figure 7), 140-wire breaks in the pipe generated 0.167 and 0.385 membership grades of states *Moderate* and *High*, respectively. Each state (linguistic value) is represented by a triangular membership function. The MGs obtained are then analyzed based on the process described in the inference algorithm. It can be seen that only 4 rules are fired in this example. Consider the first rule on the left-side of the diagram. The firing strength 0.167, above the blue oval shape with the intersection mark, is multiplied by the rule weight, 0.857, producing a weighted firing strength, 0.143, that modifies the output fuzzy set "*Poor*" of the rule. The four fired rules are evaluated in the same way and their output fuzzy sets are then aggregated to get the fuzzy set of MG contributions to the concept, $(0/Excellent,\ 0/Good,\ 0/Moderate,\ 0.321/Poor,\ 0.215/Critical)$. Consequently, the concept ***PWC*** of the pipe would be represented by two contiguous condition states, "*Poor*" and "*Critical*" with a degree of support 0.321 and 0.215, respectively, as shown in Figure 7. The output fuzzy set of MGs could be defuzzified into a single crisp value using an appropriate defuzzification method. Obviously, getting the output in either crisp or fuzzy format depends on the application but it is worth noting that defuzzification into a crisp format loses some information about the condition that might be helpful for further actions.

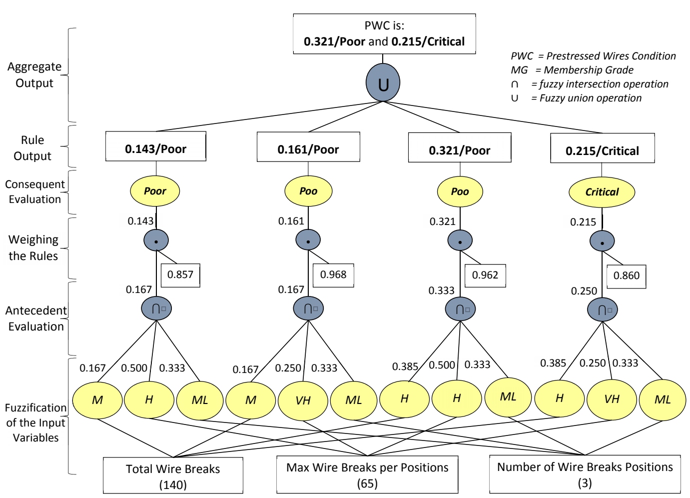

Fig. 7. Illustration of how knowledge is processed to calculate the concept *PWC* based on the fuzzy inference process presented in this paper. The crisp values of input variables are shown in the brackets, while MGs produced during processes appear next to vertical and inclined lines. The yellow oval shapes represent the states (membership functions of linguistic values) associated with the variables and concept, and their acronyms shown are: M=Moderate, H=High, VH=Very High and ML=Multiple. The smaller blue ovals represent the fuzzy logical operations performed in the inference process, as labeled on the shape.

V. Model Application and Results

The proposed fuzzy methodology has been applied to real-world data from the Man-Made River Project (MMRP) in Libya, one of the world's largest water supply projects that uses large diameter PCCP pipes, to estimate pipe condition based on the structure presented in Figure 2. For validation purposes, two models have been constructed. The first model was constructed based on the full fuzzy environment method presented in the paper. The three methods of estimating the normalized fuzzy weights based on FAHP approach indicated in the fuzzy rules construction algorithm have been applied to calculate the sets of normalized fuzzy weights for variables and concepts. The mean relative fuzziness of the weights (FuW_i) were calculated and compared, as shown in Figure 8. It has been concluded that the *row means with geometric normalization method* is the most appropriate for representing the fuzzy pairwise comparison ratios for all sets of variables and concepts because it gives almost the same relative fuzziness of the comparison ratios, which means that it preserves the fuzziness that the experts gave in their judgments.

In the second model, we made some changes in the methodology so that we used a crisp format for variable weights that have been calculated by the conventional AHP method.

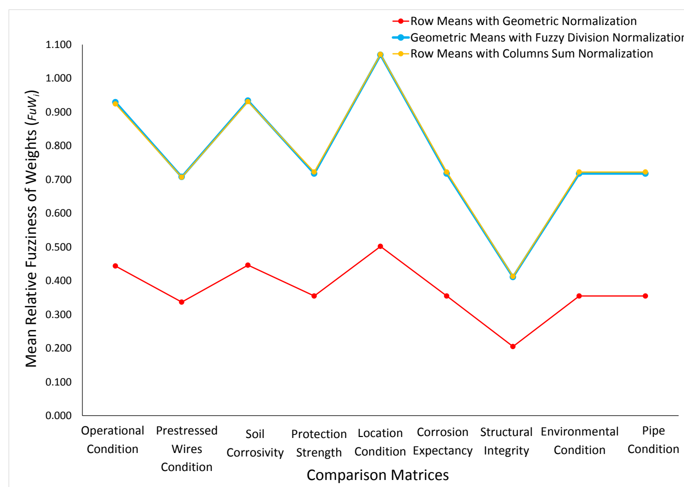

Fig. 8. Mean relative fuzziness of weights calculated by three methods (FuW_i). The method *row means with geometric normalization* gives almost the same relative fuzziness of pairwise comparison data.

The two models have been applied to a sample of 100 records of data and their outcomes were compared to actual condition outcomes (i.e. *Excellent*, *Good*, *Moderate*, *Poor* or *Critical*) estimated by a domain expert. False Positive and False Negative analyses have been carried out in order to estimate the performance of each model. In this paper, we set the following definitions for false positive and false negative:

False positive: is when the outcome condition of the model is worse than the actual condition outcome (i.e. the model gives pessimistic or overestimated results), and

False negative: is when the outcome condition of the model is better than the actual condition outcome (i.e. the model gives optimistic or underestimated results).

Figure 9 shows a summary of the comparisons between actual and model outcomes. The most important to note is the relatively high number of false negatives (i.e. 20 outcomes out of the total 100) yielded by the second model that uses the crisp weights. Further examination of the results found that all 20 outcomes are classified as *Critical* by the expert, which the second model failed to predict. This may be due to the amount of information lost by using crisp weights. False negatives are critical because they mean the model is underestimating the poor quality of pipes. An estimated pipe condition of *Good* when it is really *Poor* or *Critical* could lead to a sudden failure of the pipe with serious consequences (e.g. repair costs, environment damage, water interruption, loss of services). The false positive outcomes have less effect because overestimating poor condition simply leads to the cost of an extra inspection and monitoring.

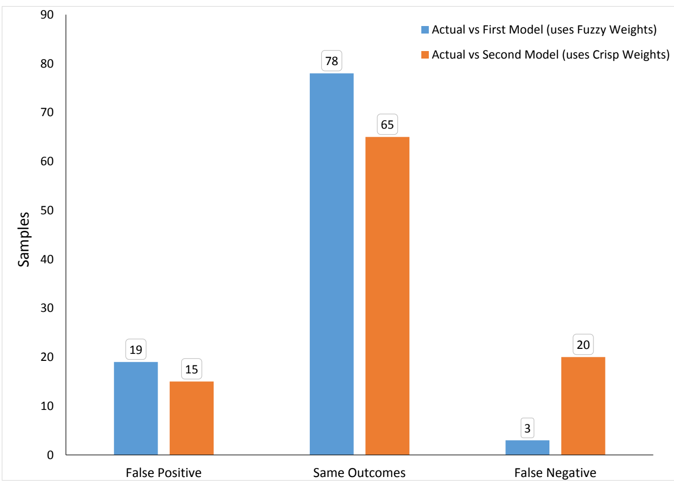

Fig. 9. Actual vs models outcomes. The blue bars represent the first model that uses triangular fuzzy weights and the orange bars represent the second model that uses crisp weights.

VI. Conclusion and Future Work

This paper has described a new methodology for eliciting and processing human expertise using a hierarchical fuzzy rule-based model. It employs fuzzy numbers for transforming and combining experts' opinions where fuzziness is maintained throughout the whole process, such that there is no loss of uncertainty information. In addition, weights of input variables are automatically incorporated into the rule so that they carry through to the inference network and allow for better uncertainty propogation through the system. The paper showed that the use of fuzzy or crisp weights leads to different results, with a better model performance in case of employing fuzzy weights.

In high-dimensional systems, a large number of rules might be required to produce accurate results. Eliciting the rules directly from experts in such cases would be a very difficult task, if not impossible. The advantage of the proposed fuzzy rule elicitation method is that knowledge engineering is not directly generating rules from experts but is providing the expertise that enables the system to generate the rules itself: rule complexity is now independent of the experts and does not constitute such a problem for them.

Future work will include testing the proposed model for generic efficacy by applying it to mental-health risk evaluation. It is a suitable domain for testing because the input data are based on human judgments as well as more objective measurements and the system's inherent uncertainty is higher. If the proposed research produces fuzzy methods that can consistently improve automated decision making, it will be an important step in validating and exploiting human expertise for decision support systems in these types of knowledge domains.

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